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Renormalization of background fields in the near vicinity of wormholes due to interaction of second order scalar and gravitational fields

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1. Introduction

During the past four-five years there has been an obvious interest in the theory of wormholes, which could possibly provide an explanation to the long-lasting and fundamental problem about the vanishing of the cosmological constant and the modification of coupling constants in quantum gravity. Wormholes can be defined as microscopic connections between smooth, large and distant regions of space-time, appearing as a result of quantum gravitational fluctuations of space-time topology [1]. It has been argued that in the process of pinching off wormholes carry away information, which becomes inaccessible to a macroscopic observer at infinity, i. e. loss of quantum coherence is observed [2,3]. A typical example for quantum coherence loss is the process of splitting up of a particle into two particles near the horizon of a black hole [4—6]. The information from the particle, which has fallen into the hole will be lost for an observer at infinity and therefore he will measure a mixed state rather than a pure one. Coherence loss has also been advocated by S. Hawking and R. Laflamme [7] in reference to the problem about nonrenormalizability of gravity due to the infinite number of effective interactions with unpredictable strengths. This effect is supposed to be significant for scalar particles [8].

However, an opposite point of view also exists — baby universes and wormholes do not cause an effective (observable) loss of quantum coherence [9],since a sequence of measurements rapidly collapses the wave function of the family of universes into one of an infinite number of coherent α eigenstates. The same opinion, although given a different interpretation, has been supported also by S. Coleman [10] — coherence has never been lost, because the different phases between the different α eigenstates remain unobservable even after a sequence of interactions.

Another important aspect of wormholes is that they turn all space-time coupling constants, masses and the cosmological constant into dynamic α variables [11,15] and thus the vanishing of the cosmological constant and the smallest possible α - value of the Newton's gravitational constant are strongly favoured [16].

In the strict mathematical sense, the insertion of wormhole ends in the baby universe is accounted by additional terms $L_i = L_i(\Phi, \partial_\mu \Phi)$ in the effective Lagrangian of wormhole theory [2,3,9—11];

(1) $$L_{eff} = L_0 + \sum \left(a_i + a_i^+\right) L_i,$$

where $L_0 = L_0(\Phi, \partial_\mu \Phi)$ and $L_i = L_i(\Phi, \partial_\mu \Phi)$ are local functions of the background scalar and gravitational fields, a_i^+ and a_i are respectively creation and annihilation operators for baby universes and i^* is the PCT transform of i.

It has been suggested in the present paper that the interaction between a fluctuating second order self interacting ($\lambda\Phi^4$) scalar field and a perturbed second order gravitational field may lead to the appearance of the above mentioned additional terms in the effective wormhole Lagrangian (1), if the short distance limit $r \to 0$ (i. e. the near vicinity of the wormhole) is considered. An important aspect of the proposed theory here is that the scalar field is assumed to be time and coordinate dependent and thus "senses" the metric perturbations of the baby universe due to the presence of the wormhole. Similar ideas have been suggested also in ref. [12,13], where in the first approximation wormholes have been treated as spherical perturbations of the background three-metric Ω_{ij}:

(2) $$g_{\mu\nu} = \sigma^2 a^2 \left(\Omega_{\mu\nu} + h_{\mu\nu}\right),$$

where a is the scale factor and $h_{\mu\nu}$ denotes the metric perturbation, usually expanded in scalar, vector and tensor harmonics on the three-sphere [14].

The present paper is organized as follows:

In Section 2 the scalar and gravitational fields have been decomposed into background ones (Φ_0, $g_{\mu\nu}$) and fluctuating ones (Φ', $h^{\mu\nu}$):

(3) $$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \text{ and } \Phi = \Phi_0 + \Phi',$$

where we assume that the fluctuation variables are of second-order and the metric perturbation varies inversely proportional to the space distance, i. e. $h_{\mu\nu} = O\left(\frac{1}{r}\right)$. For simplicity spherical harmonics decomposition has not been taken into account. The well-known quantum-gravity partition functional integral Z over all space-time geometries and matter (in the present case scalar) fields has been defined, supplemented also by an integration over the fluctuating variables (Φ', $h_{\mu\nu}$) and thus accounting for the presence of wormholes.

In Section 3 the method of zeta-function regularization has been applied for calculating the determinant of a second-order differential operator, obtained after performing the integration over the fluctuating scalar field variable Φ' in the partition functional integral Z. Calculations have been performed in the short-distance limit $r \to 0$ and also under the assumption that background fields do not change in space and time so rapidly if compared with the fluctuating fields. That is why background fields can be considered "frozen" in space and time.

In Section 4 a brief discussion of the physical meaning of the additional nonpolynomial term

$$L_{\text{ren}}\left(g_0^{\mu\nu}, \Phi_0\right) = \frac{\delta}{2}\left[-\frac{3\gamma_2\lambda\Phi_0^2}{3\gamma_1\left(m_1^2 + 6\lambda\Phi_0^2\right)}\right]^{\frac{3}{2}} \tag{4}$$

has been performed. This term, obtained in the effective wormhole Lagrangian of background fields after performing the Φ' integration, evidently accounts for the interaction between the fluctuating fields near the wormhole and in this way "signals" of the presence of the wormhole itself. In order to see whether quantum coherence will be lost or not it has to be checked whether the additional nonpolynomial term will persist at large distances (in the limit $r \to \infty$), where a distant observer is able to make certain conclusions. However, for that purpose the self-consistent problem about the space (and time) evolution of the background scalar and gravitational fields has to be solved, which is not very easy to deal with at all. Since the result obtained evidently depends on the approximation, in which it has been worked out, to a certain extent it confirms John Preskill's conclusion [16] about quantum indeterminacy of coupling constants in quantum gravity, as far as the influence of short distance physics is concerned.

2. Quantum gravity partition functional integral in the presence of fluctuating scalar and gravitational fields

Our starting point is the effective action of self-interacting ($\lambda\varphi^4$) scalar field, coupled to gravity:

$$S = \int_M d^4x \frac{1}{2}\sqrt{g}\left[-M_{pl}^2 R + g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi + \left(m^2 + \xi_0 R\right)\varphi^2 + \lambda\varphi^4\right]. \tag{5}$$

The first term in parenthesis in (5) is the gravitational part of the action, ξ_0 is a numerical coefficient, expressing the coupling between scalar and gravitational fields and the integration is performed over the closed four-manifold M of the baby universe.

As already mentioned in the Introduction, the gravitational and scalar filds are decomposed into background $\left(\hat{g}^0_{\mu\nu}, \hat{\Phi}_0\right)$ and fluctuating ones $\left(\hat{h}_{\mu\nu}, \Phi'\right)$:

$$\varphi = \hat{\Phi}_0 + \Phi', \quad \hat{g}^{\mu\nu} = \hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}, \tag{6}$$

where for convenience we have used the 'hat' variables:

$$\hat{g}_0^{\mu\nu} \equiv \sqrt{g}\,g^{\mu\nu}, \quad \hat{h}^{\mu\nu} \equiv \sqrt{g}\,h^{\mu\nu}, \quad h^{\mu\nu} = O\left(\frac{1}{r}\right) \tag{7}$$

instead of the conventional ones $g^{\mu\nu}$ and $h^{\mu\nu}$. We assume also that the signature of the background metric

$$ds^2 = dt^2 + a^2(t)\,d^3\Omega_3 \tag{8}$$

in positive (+, +, +, +). In (8) $a^2(t)$ is the scale factor and Ω_3 — the unit three-sphere.

In terms of all background and fluctuating variables, the quantum gravity partition functional integral can be defined in the following way:

$$(9) \qquad Z = \int_M d\hat{g}_0^{\mu\nu} d\hat{\Phi}_0 d\Phi' d\hat{h}^{\mu\nu} e^{-S\left(\hat{g}^0_{\mu\nu} + \hat{h}_{\mu\nu},\, \hat{\Phi}_0 + \Phi'\right)},$$

where $S\left[\hat{g}^0_{\mu\nu} + \hat{h}_{\mu\nu}, \hat{\Phi}_0 + \Phi'\right]$ is the action (5) in terms of the decomposition (6).

According to Hawking's definition [17,18], the integral (9), if taken only in respect to the background variables, gives the transition amplitude:

$$(10) \qquad Z = \left\langle \hat{\Phi}_0^{(2)}, g_{\mu\nu}^{(0)(2)} \middle| \hat{\Phi}_0^{(1)}, g_{\mu\nu}^{(0)(1)} \right\rangle$$

to go from a three-geometry $g_{\mu\nu}^{(0)(1)}$ on an initial (baby universe) spacelike surface to a three-geometry $g_{\mu\nu}^{(0)(2)}$ on a final (baby universe) spacelike surface and the integral is taken over all four geometries and scalar fields, which match $\left(\hat{\Phi}_0^{(1)}, g_{\mu\nu}^{(0)(1)}\right)$ and $\left(\hat{\Phi}_0^{(2)}, g_{\mu\nu}^{(0)(2)}\right)$ on the initial and final surfaces respectively. Unlike Hawking's definition, where no fluctuation variables are taken into account, the proposed new definition (9) of the modified partition functional integral encompasses also the short distance effects near the wormhole, which will be further investigated. Note also that a complex rotation $t \to -i\tau$ of the time coordinate has been performed so that the path integral (9) does not oscillate and does not converge.

By use of (6) and (7) the action can be decomposed into several parts, some of them containing both background and fluctuation variables:

$$(11) \qquad S\left(\hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}, \hat{\Phi}_0 + \Phi'\right) = \hat{S}_I\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}_0\right) + S_I\left(\hat{h}^{\mu\nu}, \hat{\Phi}_0, \Phi'\right) + \hat{S}_1\left(\Phi', \hat{g}_0^{\mu\nu}\right)$$

$$+ S_2\left(\left(h^{\mu\nu}\right)^2\right) + S_2\left(\hat{\Phi}_0, \left(h^{\mu\nu}\right)^2\right) + S_2\left(\hat{\Phi}_0, \Phi', \left(h^{\mu\nu}\right)^2\right) + S_2\left(\Phi', \left(h^{\mu\nu}\right)^2\right).$$

Terms $S_2(....)$ account for second-order metric perturbations and terms, containing both second-order metric and scala field perturbations account for scalar particle-graviton interactions. This is unlike the case investigated in Hawking's paper [19], where the performed action decomposition (in terms of our notations)

$$(12) \qquad S\left(\hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}, \hat{\Phi}_0 + \Phi'\right) = S_0\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}_0\right) + S_1\left(h^{\mu\nu}\right) + S_1\left(\Phi'\right)$$

in practice excludes any such interactions.

We will denote the integral over the $\left\{\hat{g}_0^{\mu\nu}\right\}$, $\left\{\hat{h}^{\mu\nu}\right\}$ and $\left\{\hat{\Phi}_0\right\}$ fields in (9) by Z_I and we will focus only on the evaluation of the Φ' integral:

$$(13) \qquad Z = Z_1 \int d\Phi' e^{-\left[S_1\left(\hat{h}^{\mu\nu}, \hat{\Phi}_0, \Phi'\right) + S_1\left(\Phi', g_{\mu\nu}^{(0)}\right) + S_2\left(\hat{\Phi}_0, \Phi', \left(h^{\mu\nu}\right)^2\right) + S_2\left(\Phi', \left(h^{\mu\nu}\right)^2\right)\right]}.$$

We shall also assume that the fluctuating scalar field changes more rapidly in comparison with the background fields so that the latter may be considered "frozen" in space and time, i. e. stationary and uniform in space. After calculating the different terms in the action (13) and rearranging the different kinds of terms, containing Φ', the following expression can be obtained:

$$Z = Z_1 \int d\Phi' e^{-\int d^4x \left[a\Phi'^2 + b\left(\partial_\mu \Phi'\right)\left(\partial_\nu \Phi'\right) + b\left(\partial_\mu \Phi'\right)\left(\partial_\nu \hat{\Phi}_0\right) + c\Phi' \right]}, \tag{14}$$

where a, b and c are, as expected, functions of the background fields $\hat{\Phi}_0$ and $\hat{g}_0^{\mu\nu}$ and also of the perturbed gravitational field $\hat{h}^{\mu\nu}$:

$$a = \left(\hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}\right)\left(R_{\mu\nu}^{(0)} + R_{\mu\nu}^{(1)}\right)\xi_0 + m^2\left(\sqrt{g^0} + \frac{1}{2}\hat{h}\right) + \frac{1}{2}\left(\sqrt{g^0} + \frac{1}{2}\hat{h}\right) + 3\lambda\hat{h}\hat{\Phi}_0^2 + 3\lambda\hat{h}^{\mu\nu}h_{\mu\nu}\hat{\Phi}_0^2, \tag{15}$$

$$b = \hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}, \tag{16}$$

$$c = 2\xi_0\hat{\Phi}_0\left(\hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}\right)\left(R_{\mu\nu}^{(0)} + R_{\mu\nu}^{(1)}\right) + 4\lambda\hat{\Phi}_0^3\left(\sqrt{g^0} + \frac{1}{2}\hat{h}\right) + m^2\hat{h}^{\mu\nu}h_{\mu\nu}\hat{\Phi}_0 + \xi\hat{h}^{\mu\nu}R_{\mu\nu}^{(1)}\hat{\Phi}_0 + 2\lambda\hat{h}^{\mu\nu}h_{\mu\nu}\hat{\Phi}_0^3. \tag{17}$$

In deriving (14) we have neglected terms, higher than second order in Φ'. After performing the integration by parts in (14), the partition functional integral Z can be written in the following way:

$$Z = Z_1 \int d\Phi' e^{-\int d^4x \left[\Phi' \hat{A} \Phi' + \Phi' \hat{B}\right]}. \tag{18}$$

In (18) $\hat{A}$ and $\hat{B}$ are differential operators of the kind:

$$\hat{A} = a - b\Box - \left(\partial_\mu b\right)\partial_\nu = a - \left[b\partial_\mu + \partial_\mu b\right]\partial_\nu, \tag{19}$$

$$\hat{B} = c - b\Box\hat{\Phi}_0 - \left(\partial_\nu\hat{\Phi}\right)\left(\partial_\mu b\right) = c - \left[b\partial_\mu + \partial_\mu b\right]\partial_\nu\hat{\Phi}_0.$$

It will be proved in the Appendix A that by a suitable gauge transformation in respect to the space variable r

$$\partial'_\mu = \partial_\mu + f(r) \tag{20}$$

the first order derivatives in (19) can be removed and therefore $\hat{A}$ and $\hat{B}$ can be written as:

$$\hat{A} = a' - b\Box', \tag{21}$$

$$\hat{B} = c - b\Box\hat{\Phi}_0. \tag{22}$$

Note that to the positive signature of the metric the Delambertian $\Box$ is:

$$\Box' = \frac{\partial^2}{\partial t^2} + \Delta'^{(3)}, \tag{23}$$

where $\Delta'^{(3)}$ is the Laplacian of the space-three-manifold, embedded in a given three-geometry. In other words, the choice of the variable r will not affect the eigenvalue spectrum of the operators $\hat{A}$ and $\hat{B}$, since the background fields (and thus the functions a, b and c, depending on them) are "frozen" in space and time according to our previous assumption.

3. Zeta-function regularization and the resulting scalar-gravity background field renormalization

Our main aim in this Section will be the evaluation of the integral (18). First, we make the simple transformation:

$$\Phi' = \Phi_1' - \frac{1}{2}\hat{B}\hat{A}^{-1} \tag{24}$$

and thus (18) can be written in a more familiar way:

$$\begin{aligned} Z &= Z_1 e^{-\frac{1}{4}\hat{B}^2\hat{A}^{-2}+\frac{1}{2}\hat{B}^2\hat{A}^{-1}} \int d\Phi_1' e^{-\Phi_1'\hat{A}\Phi_1'} \\ &= Z_1 e^{\frac{1}{2}\hat{B}^2\hat{A}^{-1}-\frac{1}{4}\hat{B}^2\hat{A}^{-2}} \left(\det \hat{A}\right)^{-\frac{1}{2}}. \end{aligned} \tag{25}$$

Note that the transformation (24) can still be applied no matter that $\hat{A}$ and $\hat{B}$ are not functions, but operators.

The determinant of the differential operator $\hat{A}$ will be calculated by applying the well-known method of zeta-function regularization, previously developed by Hawking [20] and others [21,22]. For this purpose the following basic formulae will be used:

$$\det \hat{A} = \exp\left[-\left(\frac{d\zeta(s)}{ds}\right)_{s=0}\right], \tag{26}$$

where

$$\zeta(s) = \sum \lambda_n^{-s} \tag{27}$$

is the defined in [20] generalized zeta-function and λ_n are the eigenvalues of the differential operator $\hat{A}$, s is an integer number ($s>2$). Since we intend to reduce our problem to a well-known quantum-mechanical problem, we will assume that the space-three manifold, over which the operator $\hat{A}$, is defined, is the three-sphere. Of course, the eigenvalues can also be found in the case of other manifolds.

In the present case we denote the eigenvalues of the operator by $\hat{A}$ by $\lambda_n = E_n^2$. This means that the operator equation

$$\hat{A}\Psi = 0 \tag{28}$$

is being satisfied by an arbitrary function Ψ of the kind:

(29) $$\Psi = e^{E_n t} f_n(r).$$

The last also means that only stationary states of (28) will be investigated. Substituting (29) into (28), the following equation for the eigenvalue functions f_n is obtained:

(30) $$\Delta^{(3)} f_n(r) = \left[-E_n^2 + V\left(r,\, \hat{\Phi}_0,\, \hat{g}^0_{\mu\nu}\right)\right] f_n(r)$$

where $\Delta^{(3)}$ is the Laplacian on the three-sphere (i. e. in spherical coordinates r, θ and φ):

(31) $$\Delta^{(3)} \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\varphi^2}.$$

The function $V\left(r,\, \hat{\Phi}_0,\, g^0_{\mu\nu}\right)$ can be evaluated in the short-distance limit $r \to 0$ by use of (15) — (17), (21), (22) and the following estimates for the gravity field [23]:

(32) $$\hat{h}^{\mu\nu} \equiv \sqrt{g} h^{\mu\nu} = O\left(\frac{1}{r}\right);\ \frac{\partial h^{\mu\nu}}{\partial x^{\lambda}} = O\left(\frac{1}{r^2}\right);\ R^{(1)}_{\mu\nu} \sim \frac{\partial^2 h^{\mu\nu}}{\partial x^{\lambda} \partial x^{\rho}} = O\left(\frac{1}{r^3}\right).$$

Keeping in mind the above assumptions, the following expression in the limit $r \to 0$ is derived:

(33) $$V\left(r,\, \hat{\Phi}_0,\, \hat{g}^{(0)}_{\mu\nu}\right) = \alpha R^{(0)}_{\mu\nu}\xi_0 + \frac{1}{2}\gamma_1\left(m_0 + 6\lambda\hat{\Phi}_0^2\right) + \frac{1}{r}\left(3\gamma_2\lambda\hat{\Phi}_0^2\right) + \frac{1}{r^2}\left(\xi_0\beta_1\hat{g}_0^{\mu\nu}\right) + \frac{1}{r^3}\xi_0\beta_2,$$

where γ_1, γ_2, γ_3, are constants. The function (33) may physically be interpreted as a three-dimensional potential barrier of the kind:

(34) $$V(r) = A_1 + \frac{B_1}{r} + \frac{C_1}{r} + \frac{D_1}{r^3};\ A_1,\, B_1,\, C_1,\, D_1 \text{ are constants}$$

which will evidently affect the eigenvalues of the operator $\hat{A}$. It may be thought as if in the vicinity of their neck ($r \to 0$) wormholes create an effective potential barrier, influencing scalar (and other) particles, penetrating down the wormhole from the baby universe. Since in a higher approximation the perturbed metric may involve higher powers of $\frac{1}{r}$, we should keep in mind the simple fact from quantum mechanics [24], that the motion of a particle in a potential field $U = -\frac{\alpha}{r^s}(s > 2)$ is restricted in a small area around the coordinate center and will finally fall upon it. However, such a movement is unlikely in the present case because of space-time nonlocality (Fig. 1).

Moreover, such a restricted motion would contradict current models in quantum gravity, based on the assumption about infinite past and infinite future states on different space surfaces [25]. It can be therefore concluded that a particle, falling down the wormhole will probably "sense" only the lower order terms and ($\sim \frac{1}{r}$ and $\sim \frac{1}{r^2}$)

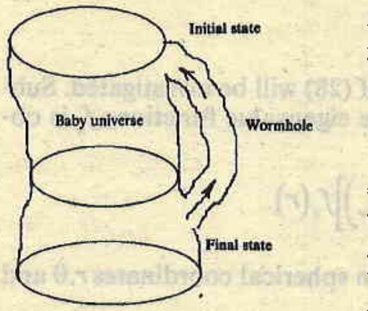

Fig. 1. Particle motion down the wormhole from an initial state to a final state and thus not crossing the intermediate state of the baby universe [25]

of the potential barrier. However, the following question inevitably arises: if the fluctuating metric involves higher order terms of $\frac{1}{r}$, how can this process of "sensing" happen? A suggestion is made in this paper that in a more realistic model the background scalar and gravitational fields continuously evolve in a self-consistent manner and thus changing the coefficients A_1, B_1, C_1, D_1, unlike in our simplified model. By means of a such self-regulating mechanism the potential terms are adjusted in such a way so that falling particles are influenced only by lower order potential terms in $\frac{1}{r}$. That is why we shall neglect terms in the fluctuating metric with higher powers than $O\left(\frac{1}{r^2}\right)$.

Further, we present the eigenvalue function $f_n(r)$ as:

$$f_n(r) = \chi_n(r) Y_{lm}(\theta, \varphi), \tag{35}$$

where $Y_{lm}(\theta, \varphi)$ are the usual spherical functions. We have to keep in mind also that in a spherically symmetric field an additional "centrifugal" term $\frac{l(l+1)}{r^2}\chi_n$ appears in the potential as a result of moment conservation and level degeneracy [24]. From (30), (31), (33) and (35) the following equation for $\chi_n(r)$ is obtained:

$$\frac{d^2\chi_n(r)}{dr^2} + \frac{2}{r}\frac{d\chi_n(r)}{dr} + \left[E_n^2 - A_1 - \frac{B_1}{r} - \frac{C_1}{r^2} - \frac{l(l+1)}{r^2}\right]\chi_n = 0, \tag{36}$$

where

$$A_1 = \alpha R^{(0)}_{\mu\nu} + \frac{1}{2}\left(m^2 + 6\lambda\hat{\Phi}_0^2\right)\gamma_1, \tag{37}$$

$$B_1 = 3\gamma_2 \lambda \hat{\Phi}_0^2, \tag{38}$$

$$C_1 = \xi_0 \beta \hat{g}_n^{\mu\nu} \tag{39}$$

and l can be regarded as the conserved momentum of the baby universe. In terms of the new variables

$$\rho = 2r\sqrt{E_n^2 - A_1}, \tag{40}$$

$$n_1 = \frac{B_1}{2\sqrt{A_1 - E_n^2}}, \tag{41}$$

$$c_1 + l(l+1) = l_1(l_1 + 1) \tag{42}$$

equation (36) can be written as:

$$\frac{d^2\chi_n(\rho)}{d\rho^2}+\frac{2}{\rho}\frac{d\chi_n(\rho)}{d\rho}+\left[\frac{1}{4}-\frac{n_1}{\rho}-\frac{l_1(l_1+1)}{\rho^2}\right]\chi_n(\rho)=0. \tag{43}$$

It is reasonable to search out for a solution, which is finite at $\rho\to 0$ (of the form $\chi_n\sim\rho^{l_1}$) and which is vanishingly small at $\rho\to\infty$ (of the form $\chi_n\sim e^{-\frac{\rho}{2}}$) [24]. Both assumptions are physically reasonable, since all fields decay at infinity and it is useless to deal with infinite solutions at $\rho\to 0$. Therefore, the function $\chi_n(\rho)$ can be written as:

$$\chi_n(\rho)=e^{-\frac{\rho}{2}}\rho^{l_1}w(\rho). \tag{44}$$

Substituting (44) into (43), we derive the equation

$$\rho w''+(2l_1+2-\rho)w'+(n_1-l_1-1)w=0, \tag{45}$$

where the prime denotes the derivative in respect to the variable ρ. Equation (45) has a well-known solution [24]:

$$w(\rho)=F(-n_1+l_1+1,2l_1+2,\rho), \tag{46}$$

where F is the so called degenerate hypergeometric function:

$$F(\alpha,\gamma,z)=\frac{\Gamma(\gamma)}{2\pi i}\int_C e^t(t-z)^{-\alpha}t^{\alpha-\gamma}dt=1+\frac{\alpha}{\gamma}\frac{z}{1!}+\frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{z^2}{2!}+\ldots \tag{47}$$

The contour C comes from infinity (when Re $t\to-\infty$), goes to infinity (when Re $t\to+\infty$) and has poles at $t=0$ and $t=z$. It is evident from (47) that the solution $F(\alpha,\gamma,z)$ is vanishing at infinity when $\alpha\le 0$. In the present case $\alpha=-n_1+l_1+1$ and so the following condition has to be fulfilled:

$$n_1\ge l_1+1 \tag{48}$$

This means that the eigenvalue levels are resticted from below. From (37), (38) and (41) the eigenvalue spectrum $\lambda_n=E_n^2$ can easily be found:

$$\lambda_n=E_n^2=\frac{1}{2}\left(m_1^2+6\lambda\hat{\Phi}_0^2\right)-\frac{3\gamma_2\lambda\hat{\Phi}_0^2}{4n_1^2}, \tag{49}$$

where m_1 is the renormalized mass due to the action of the background gravitational field:

$$m_1^2=m^2+2\alpha R_{\mu\nu}^{(0)}. \tag{50}$$

It can easily be checked that the spectrum (49) is always positive in the limiting case $\frac{\lambda\hat{\Phi}_0^2}{m_1^2}\ll 1$. In the opposite case ($\frac{m_1^2}{6\lambda\hat{\Phi}_0^2}\ll 1$) E_n can be positive if the inequality

$$n_1^2\gg\frac{\gamma_2}{3\lambda\hat{\Phi}_0^2} \tag{51}$$

is satisfied. If we assume that γ_2 and λ are positive constants, this may really happen, since according to (48) the eigenvalue levels are restricted from below and the investigated case $\frac{m_1^2}{6\lambda\hat{\Phi}_0^2} \ll 1$ requires $\lambda\hat{\Phi}_0^2$ to be great. We will also assume that the positive (and thus continuous) eigenvalue spectrum ranges from zero to infinity. From (39), (42), (51) and (48) it is evident that such an assumption is justifiable especially for small l and small constants ξ_0 or β.

Finally, the generalized zeta-function (27) can be calculated, replacing the sum over n_1 by a continuous integration (since the spectrum is continuous) and multiplying each $\lambda_n = E_n^2$ from (49) by n^2, since each eigenvalue level with a main quantum (level) number n is $\sum_{l=0}^{n-1}(2l+1) = n^2$ degenerate:

$$\zeta(s) = \int_0^\infty dn_1 n_1^2 \lambda_{n_1}^{-s} = \int_0^\infty dn_1 n_1^2 \left[\frac{1}{2}\left(m_1^2 + 6\lambda\hat{\Phi}_0^2\right)\gamma_1 - \frac{3\gamma_2\lambda\hat{\Phi}_0^2}{4n_1}\right]^{-s}. \tag{52}$$

By making the substitutions

$$K = \frac{1}{2}\gamma_1\left(m_1^2 + 6\lambda\hat{\Phi}_0^2\right), \tag{53}$$

$$L = \frac{3\gamma_2\lambda\hat{\Phi}_0^2}{4} \tag{54}$$

the integral (52) can be written as:

$$\zeta(s) = \frac{1}{K^s}\int_0^\infty dn_1 \frac{n_1^{2(s+1)}}{\left(n_1^2 - \frac{L}{K}\right)^s}. \tag{55}$$

The last integral is of well-known type and can be evaluated by means of the following formulae [26]:

$$\int_0^\infty dn_1 \frac{n_1^\beta}{\left(n_1^2 + M^2\right)^\alpha} = \frac{\Gamma\left(\frac{1+\beta}{2}\right)\Gamma\left(\alpha - \frac{1+\beta}{2}\right)}{2\left(M^2\right)^{\alpha - \frac{1+\beta}{2}}\Gamma(\alpha)}, \tag{56}$$

where Γ is the gamma-function $\Gamma(\alpha) = \int_0^\infty e^{-x}x^{\alpha-1}dx$. We obtain the following expressions for the generalized zeta-function $\zeta(s)$ and its first derivative:

$$\zeta(s) = \frac{1}{K^s}\left\{\frac{\Gamma\left(s + \frac{3}{2}\right)\Gamma\left(-\frac{3}{2}\right)}{2\left(-\frac{L}{K}\right)^{-\frac{3}{2}}\Gamma(s)}\right\} = \frac{1}{K^s}I(s), \tag{57}$$

$$\frac{d\zeta(s)}{ds} = -\frac{s}{K^{s+1}} I(s) + \frac{1}{K^s} \frac{\Gamma\left(s+\frac{3}{2}\right)}{\Gamma(s)} \left(\Psi\left(s+\frac{3}{2}\right) - \Psi(s)\right), \tag{58}$$

where *I(s)* denotes the term inside the brackets in (57). The following standard expressions have also been used:

$$\Gamma'(z) \equiv \Gamma(z)\Psi(z); \quad \Gamma(z+1) \equiv z\Gamma(z),$$

$$\Psi(n+1) = 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \gamma; \quad \Psi(z+1) = \frac{1}{z} + \Psi(z). \tag{59}$$

$\Psi(z)$ is the Oiler function and γ — the Oiler constant:

$$\gamma = \lim_{n\to 0}\left(1 + \frac{1}{2} + \ldots + \frac{1}{n} - \ln n\right) = 0{,}5772157. \tag{60}$$

The final result for the partition functional integral (25) with account of (26), (52) and (58) is

$$Z = Z_1 e^{\frac{1}{2}\hat{B}^2\hat{A}^{-1} - \frac{1}{4}\hat{B}^2\hat{A}^{-2}} \left[\exp\left(-\frac{d\zeta}{ds}\right)_{s=0}\right]^{-\frac{1}{2}} \tag{61}$$

$$= Z_1 \exp\left[\frac{1}{2}\hat{B}^2\hat{A}^{-1} - \frac{1}{4}\hat{B}^2\hat{A}^{-2}\right] \exp\left[\frac{\delta}{2}\left(-\frac{3\gamma_2}{2\gamma_1}\frac{\lambda\hat{\Phi}_0^2}{\left(m_1^2 + 6\lambda\hat{\Phi}_0^2\right)}\right)^{\frac{3}{2}}\right].$$

4. Discussion

The physical meaning of eq. (61) will be more easily revealed if we rewrite it into another way:

$$Z = \int d\hat{g}_0^{\mu\nu} d\Phi_0 \exp\left(\frac{1}{2}\hat{B}^2\hat{A}^{-1} - \frac{1}{4}\hat{B}^2\hat{A}^{-2}\right) \tag{62}$$

$$\times \exp\left\{-\int d^4x\left[L_0\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}\right) + L_{\text{ren}}\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}_0\right)\right]\right\},$$

where we will call $L_{\text{ren}}\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}_0\right)$ a renormalized (nonpolynomial) Lagrangian:

$$L_{\text{ren}}\left(\hat{g}_0^{\mu\nu}, \hat{\Phi}_0\right) = \frac{\delta}{2}\left[-\frac{3\gamma_2}{\gamma_1}\frac{\lambda\hat{\Phi}_0^2}{\left(m_1^2 + 6\lambda\hat{\Phi}_0^2\right)}\right]^{\frac{3}{2}}. \tag{63}$$

The physical essence of expression (63) in that the interaction between the fluctuating second-order gravitational and scalar fields leads to an effective renormalization of the background fields with an additional nonpolynomial non-local term. The numerical constant δ in the renormalization term (63) and (61) equals to

$$\delta = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(-\frac{3}{2}\right)}{2\Gamma(0)}\left\{\Psi\left(\frac{3}{2}\right) - \Psi(0)\right\} \tag{64}$$

~and has an important physical meaning. Remember that δ has appeared in the process in the evaluation of the zeta-function integrals (52), (55), originally derived from equation (65):

$$\Delta^{(3)} f_n(r) = \left[-E_n^2 + V\left(r, \hat{\Phi}_0^2, \hat{g}_0^{\mu\nu}\right)\right] \tag{65}$$

with a potential barrier $V\left(r, \hat{\Phi}_0, \hat{g}_0^{\mu\nu}\right)$ (33), reflecting the properties of short distance physics ($r \to 0$) and of the background fields. That is why to a certain extent δ may be viewed as a coupling constant between short-distance physics and background fields. Note also that the self-coupling constant λ of the scalar field plays an important role in the renormalization term (63) and thus shows that the initial "nonlinear" features of the scalar field are an important prerequisite for this result. Unhappily, at the present moment it would be premature to relate the constant or its numerical value to any concrete physical observable. In this aspect it would be interesting to check whether a similar coupling constant will appear when working out in a higher order approximation of the fluctuating gravitational field. Unfortunately, it has not been possible to find an exact solution of (65), when higher order terms $\sim O\left(\frac{1}{r^3}\right)$ were included. Then it may probably happen that the additional term (63) is not unique and thus will depend on the approximation, assumed in advance. That is why the result in this paper confirms, to a certain extent, J. Preskill's conclusion in ref. [16] about quantum indeterminacy of our predictions, as far as short-distance-physics is concerned: "It is because the renormalization of G (the Newton's gravitational constant) is dominated by nonuniversal short-distance effects that we are unable in the end to make precise predictions about the values of other constants."

However, in Section 3 of this paper it was emphasized that quantum indeterminacy will not be so clearly manifested because only the lowest order terms $\sim O\left(\frac{1}{r}\right)$ and $\sim O\left(\frac{1}{r^2}\right)$ would probably be the most dominant ones, when investigating the motion of particles through the wormhole. In reference to the problem about coherence loss it is worth reminding that it may be established only by a distant observer. In the present case, however, we have not solved the self-consistent problem about scalar and gravitational field evolution and that is why we do not know whether the additional renormalization term will persist or die off at large distances. In other words, although the additional renormalization term (63) resembles in a sense the additional term (1), here we cannot affirm that global effects due to wormholes are surely induced. However, such a possibility about the existence of global effects should not be excluded until the self-consistent problem about scalar and gravitational field evolution is solved. The latter means that a system of two nonlinear differential equations should be solved. It is known that if a stable solution (or a configuration) is assumed to exist, then such a solution can be found in the framework of the so-called self-organization theory [27]. As a common rule in this theory, space nonuniform distribution of a given physical variable can give a greater variety of scenarios for its evolution.

It should be noted also that the correct treatment of the partition functional integral (9) should include some gauge-fixing terms so that (9) would be invariant under arbitrary gauge transformations of the fluctuating scalar and gravitational fields [28—30]. However, we are interested only in gravitational fields, which are asymptotically flat at infinity $\left(h_{\mu\nu} = \left(\frac{1}{r}\right)\right)$ — an assumption, frequently applied in wormhole theory. Also, we have removed all fist-order derivatives in the operators $\hat{A}$ and $\hat{B}$((19), also Appendix A), which can break the gauge invariance in respect to the fluctuating scalar field. Moreover, it is unlikely that the additional term (63) of background fields may appear as a consequence of the introduction of gauge-fixing terms and Faddeev-Popov determinants [28], depending on fluctuating field variables and not on the background field variables.

As a final result of this paper, it may be concluded that wormholes (and fluctuating topologies in general) may possibly "amplify" and change the surrounding scalar and gravitational fields. This can be thought as a quantum gravity and topology effect and is worth further investigating it, despite our present ignorance about the essence of this phenomena.

5. Appendix A

It shall be proved in this appendix that after performing a gauge (coordinate) transformation

(66) $$\partial'_{\mu} = \partial_{\mu} + f(r)$$

the operators $\hat{A}$ and $\hat{B}$ can be written as second order operators only and in terms of the new space variable r' in (21), (22):

(67) $$\hat{A} = a' - b'\Box' ; \quad \hat{B} = c - b'\Box' \hat{\Phi}_0.$$

in other words, in accordance with the formulas (19)

(68) $$\hat{A} = a - b\Box - \left(\partial_{\mu} b\right)\partial_{\nu} = a - \left[b\partial_{\mu} + \left(\partial_{\mu} b\right)\right]\partial_{\nu},$$

$$\hat{B} = c - b\Box\hat{\Phi}_0 - \left(\partial_{\nu}\hat{\Phi}_0\right)\left(\partial_{\mu} b\right) = c - \left[b\partial_{\mu} + \left(\partial_{\mu} b\right)\right]\left(\partial_{\nu}\hat{\Phi}_0\right)$$

we will have to establish the validity of the following operator equality:

(69) $$\left[b\partial_{\mu} + \left(\partial_{\mu} b\right)\right]\partial'_{\nu} = b\partial'_{\mu}\partial'_{\nu}$$

which has to be satisfied for any gauge transformation of the kind (66). After substituting (66) into (69) we obtain

(70) $$b\partial_{\mu}\partial_{\nu} + \left(\partial_{\mu} b\right)\partial_{\nu} = b'\left(\partial_{\mu} + f(r)\right)\left(\partial_{\nu} + f(r)\right).$$

In (70) and (69) b' is a function of the new variable r'. However, according to the equivalence principle in gravity theory the relative order dependence $h_{\mu\nu} = O\left(\frac{1}{r}\right)$ in the function

$$b = \hat{g}_0^{\mu\nu} + \hat{h}^{\mu\nu}$$

should hold in any coordinate system. That is why it can be written:

$$b'(r') = b(r) = O\left(\frac{1}{r}\right) \sim \frac{\alpha}{r}; \quad \alpha = \text{const} \tag{71}$$

and therefore the first terms on the left and the right side of (70) cancel each other. Furthermore, let us assume that (70) acts on an arbitrary (but known) function $g(r)$. As a result we derive:

$$\left[(\partial_\mu b) - bf\right](\partial_\nu g) = \left[b(\partial_\mu f) + bf^2\right]g. \tag{72}$$

If we denote

$$\frac{\partial_\nu g}{g} = A(r) \tag{73}$$

and take into account the estimates:

$$b(r) = \frac{\alpha}{r}; \ \partial_\mu b = -\frac{\alpha}{r^2} \tag{74}$$

finally we derive the following differential equation for the function $f(r)$:

$$\frac{\alpha}{r}\frac{df}{dr} + \frac{f^2}{r^2} + \alpha\frac{A(r)}{r}f + \alpha\frac{A(r)}{r^2} = 0 \tag{75}$$

Our statement will be proved if we establish that a solution of this differential equation exists for an arbitrary function $f(r)$. However, the differential equation (75) is of the type:

$$\frac{df}{dr} = F(f, r) \tag{76}$$

and from the theory it is well-known that a solution of this equation always exists.

This precludes the proof of the statement.

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Ренормализация на фонови полета в близката околност на топологически пространствено-времеви структури (от тип „ръчки") вследствие взаимодействие на скаларни и гравитационни полета от втори порядък

Богдан Димитров

(Р е з ю м е)

В рамките на теорията за пространствено-времевите структури (от тип „ръчки") в най-ранната Вселена е доказано, че взаимодействието на флуктуационни скаларни и гравитационни полета от втори порядък води до ренормализация на действието на фоновите полета с допълнителен член от неполиномиален тип. Приложен е методът на дзета-функция регуляризация за изчисляването на неполиномиалния член, зависещ съществено от нелинейните свойства на скаларното поле ($\lambda\Phi^4$теория).

Предложена е физическа интерпретация, съгласно която структурите „ръчки" създават ефективен квантово-механичен потенциален бариер, въздействащ върху частиците, преминаващи през ръчките.